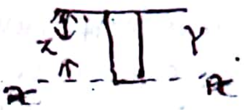
Principles of effective stress:- Geostatic stress is stress due to self wt. of soil when ground surface horizontal and the properties of soil do not change along the horizontal plane. (1)

- Concept of stress for a particular system:- [Geostatics stress in soil with horizontal surface]
  - Stress mean force per unit area.
  - vertical force applied on the plane normal to the vertical force i.e horizontal plane.
  - horizontal force applied on the vertical plane.

→ vertical stress:-
- area of cross-section of soil = (A)
- sectional distance z from surface
- bulk unit weight (γ)
- total vertical stress (σ_z) = weight of soil in considerable part / Area of base.



$$= \frac{\gamma(z \times A)}{A} = \gamma z.$$

Horizontal stress

Effective stress principle:-
- Karl Terzaghi (1936), important theory: any plane in a soil mass, the total stress or unit pressure (σ) is total load per unit area. [pressure may be due to (i) self weight of the soil and (ii) over burden in the soil.
- total pressure consists of two distinct components (i) intergranular pressure or effective pressure. (ii) neutral pressure or pore pressure.
  (i) effective pressure is the pressure transmitted from particle through their point of contact through the soil above the plane.
  (ii) pore pressure is the pressure transmitted through the pore fluid.

So, total vertical pressure = (effective pressure) + (pore pressure)

$$\boxed{\sigma = \bar{\sigma} + u}$$

∴ pore pressure is equal to peizometric head i.e. $h_w \gamma_w$.

or, $\bar{\sigma} = \sigma - u$

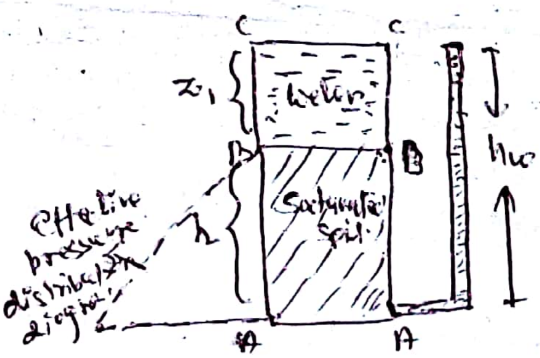
$$\boxed{\bar{\sigma} = \sigma - h_w \gamma_w}$$

Effective pressure for the submerged soil mass:-

total pressure at A-A is given by:-

$$\sigma = h\gamma_{sat} + z_1 \gamma_w$$

Now, $u = h_w \times \gamma_w = h_w \gamma_w$.



$$\bar{\sigma} = \sigma - u$$
$$= (h\gamma_{sat} + z_1\gamma_w) - h_w\gamma_w$$
or, $h\gamma_{sat} + z_1\gamma_w - (h + z_1)\gamma_w$
or, $h\gamma_{sat} + \cancel{z_1\gamma_w} - h\gamma_w - \cancel{z_1\gamma_w}$
or, $h\gamma_{sat} - h\gamma_w = h(\gamma_{sat} - \gamma_w)$
$$= h\gamma'$$

$$\therefore \bar{\sigma} = h\gamma'$$

∴ Shows:- ($\bar{\sigma}$) does not depend upon the height of water above, if z_1 reduces to zero, the ($\bar{\sigma}$) will not change (remains constant) so long as the soil mass above A-A remains fully submerged.

at B-B section:-

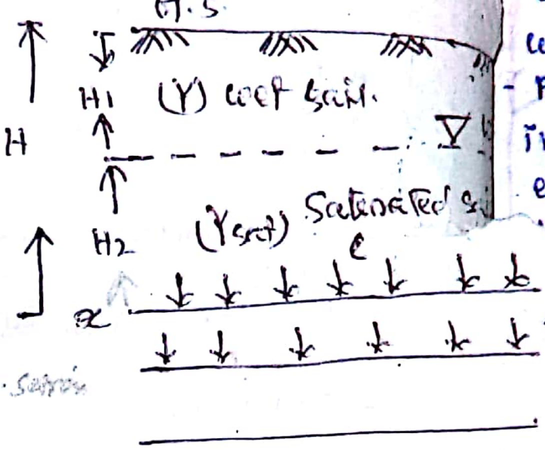
∴ total pressure is equal to water pressure only.

Now, $\sigma = \gamma_w z_1$ then, $\bar{\sigma} = \sigma - u = \gamma_w z_1 - (h_w \gamma_w) = ~~\gamma_w(z_1\gamma_w)~~$

$= \gamma_w z_1 - z_1 \gamma_w = 0$ [when inserting piezometer at this point given head (z_1)]

② Effect of water-table fluctuations on effective stress:-

- there is change in the effective stress due to fluctuation of water-table:-
- depth of water table is (H_1) below ground surface.
- Soil above the W.T. is assumed to be wet with bulk unit weight (γ)
- Soil below the water-table is ~~saturated~~ Saturated with saturated unit weight of (γ_{sat})



⇒ Let us consider any section (x-x).

- the downward force (P) at section (x-x) is equal to the weight of the soil. Thus:-

$$W = (\gamma H_1 A) + (\gamma_{sat} H_2 A)$$ where, A is area of cross-section of soil mass.

$$\sigma = \frac{W}{A} = \frac{\gamma H_1 A}{A} + \frac{\gamma_{sat} H_2 A}{A} = (\gamma H_1 + \gamma_{sat} H_2)$$

pore water pressure = $u = \gamma_w H_2$

Now, effective stress = $\bar{\sigma} = \sigma - u$

$$= (\gamma H_1 + \gamma_{sat} H_2) - \gamma_w H_2.$$

or, $\gamma H_1 + \gamma_{sat} H_2 - \gamma_w H_2$

or, $\gamma H_1 + H_2(\gamma_{sat} - \gamma_w) = [\gamma H_1 + \gamma' H_2]$

or, $\boxed{\bar{\sigma} = \gamma H_1 + \gamma' H_2}$ where, γ' = submerged unit weight.

gives value of effective stress at section (x-x)

[Note, if there would be water level (superficial) above the soil as in the case of previous example which deals with the calculation of effective stress in submerged soil, we have to also add the weight of the water in addition of the weight of wet soil and saturated soil.

condition (a) If the water table rises to the ground surface, the whole of the soil is saturated. then total stress at the soil in section (x-x) will be:-

$\sigma = (\gamma_{sat} H_1 + \gamma_{sat} H_2)$. then, $\bar{\sigma} = \sigma - u = (\gamma_{sat} H_1 + \gamma_{sat} H_2) - \gamma_w (H_1 + H_2)$

$$= \gamma_{sat}(H_1 + H_2) - \gamma_w (H_1 + H_2) = H(\gamma_{sat} - \gamma_w)$$
$$= H\gamma'$$

since, $\gamma' < \gamma$, the effective stress becomes less.

condition (b) water table depressed below the (x-x) section.

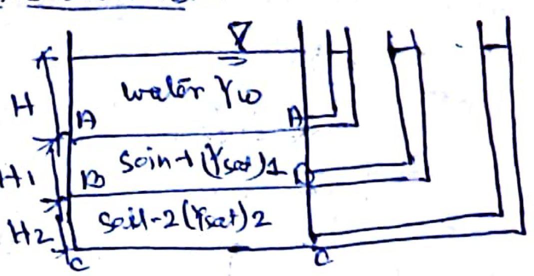
then, $\sigma = \gamma H$ because, there is not water above and thus no saturated condition.

$\bar{\sigma} = \sigma - u = \gamma H - 0$ (because no water above. so, no piezometric head)

Since, $\gamma > \gamma'$, effective stress increases.

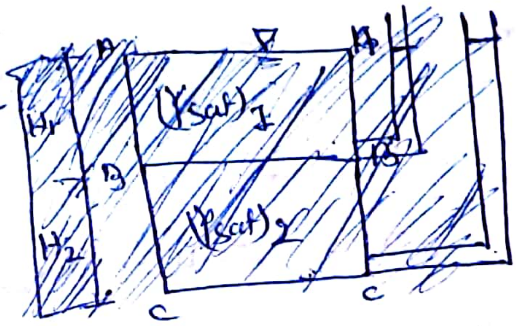
Effective stress in a soil mass under hydrostatic conditions:-

- Fig shows; a soil mass under hydrostatic conditions, wherein the water level remains constant.



- As the interstices in the soil mass are interconnected, water rises to the same elevation in different piezometers fixed to the soil mass.
- The effective stress at various sections can be determined using the of effective stress.

i.e., $\boxed{\bar{\sigma} = \sigma - u}$



① water table above the soil-surface A-A:-

Section (A-A)

$\sigma = \gamma_w H, \; u = \gamma_w H$

therefore $\bar{\sigma} = \sigma - u = (\gamma_w H - \gamma_w H) = 0.$

Section (B-B)

$\sigma = \gamma_w H + (\gamma_{sat})_1 \cdot H_1 ; \; u = \gamma_w (H + H_1)$

therefore, $\bar{\sigma} = (\sigma - u) = [\gamma_w H + (\gamma_{sat})_1 H_1] - [\gamma_w (H + H_1)]$

$= \cancel{\gamma_w H} + (\gamma_{sat})_1 \cdot H_1 - \cancel{\gamma_w H} - \gamma_w H_1$

$= (\gamma_{sat})_1 \cdot H_1 - \gamma_w H_1.$

$= H_1 (\gamma_{sat_1} - \gamma_w) = H_1 \gamma_1'$ where, (γ_1') is submerged unit weight of soil-1.

Section (C-C)

$\sigma = \gamma_w H + (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2$

$u = \gamma_w (H_2 + H_1 + H)$

$\therefore \bar{\sigma} = \sigma - u = [\gamma_w H + (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2] - \gamma_w (H_2 + H_1 + H)$

$= \cancel{\gamma_w H} + (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2 - \gamma_w H_1 - \cancel{\gamma_w H}$

$= (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2 - \gamma_w H_1$

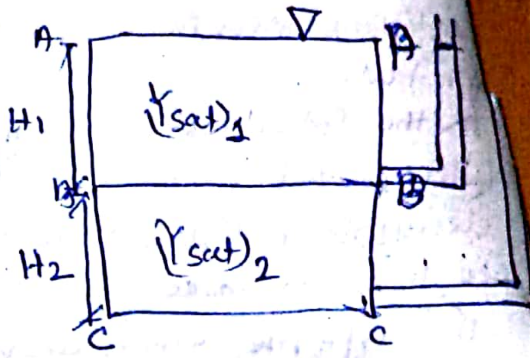
or, $(\gamma_{sat})_1 \cdot H_1 - \gamma_w H_1 + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2$

or, $H_1 [(\gamma_{sat})_1 - \gamma_w] + H_2 [(\gamma_{sat})_2 - \gamma_w]$

$\boxed{\bar{\sigma} = H_1 \times \gamma_1' + H_2 \gamma_2'}$ where, γ_2' is the submerged unit weight of soil-2

(2) water table at the soil surface A-A

- Fig. shows the condition when the depth "H" of water above the section A-A is reduced to zero.



- In this case the effective stress at various sections are determined as under:-

Section A-A

$$\sigma = u = \bar{\sigma} = 0$$

i.e $\sigma = 0,\ u = 0,\ \bar{\sigma} = 0$

Section B-B

$$\therefore \sigma = (\gamma_{sat})_1 \cdot H_1 \text{ and } u = \gamma_w H_1$$

$$\bar{\sigma} = \sigma - u = (\gamma_{sat})_1 \cdot H_1 - \gamma_w H_1$$

$$= H_1 [(\gamma_{sat})_1 - \gamma_w]$$

$$= H_1 \cdot \gamma_1'$$

where, γ_1' is the submerged unit weight of soil-1.

Section at C-C

$$\sigma = (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2, \quad u = \gamma_w (H_1 + H_2)$$

$$\therefore \bar{\sigma} = (\sigma - u) = [(\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2] - [\gamma_w (H_1 + H_2)]$$

$$= [(\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 H_2 - \gamma_w H_1 - \gamma_w H_2$$

$$= (\gamma_{sat})_1 \cdot H_1 - \gamma_w H_1 + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2$$

$$= H_1 [(\gamma_{sat})_1 - \gamma_w] + H_2 [(\gamma_{sat})_2 - \gamma_w]$$

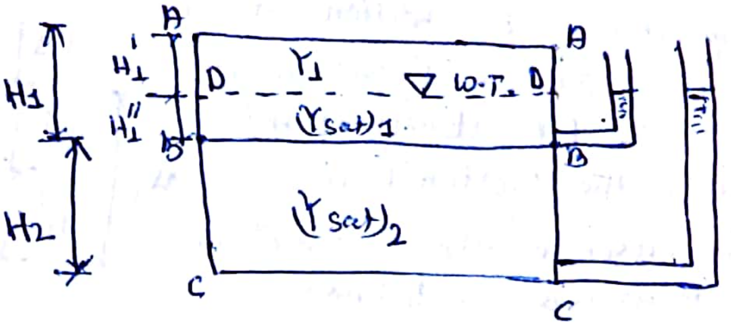
$$[\bar{\sigma} = H_1 \cdot \gamma_1' + H_2 \cdot \gamma_2']$$

where, γ_2' is the submerged unit weight of soil-2

[Above equations shows that, the effective stress in a soil mass is independent of the depth of water above the soil-surface.]

(3) Water-table in soil-1

- Fig shows the case when water table is at D-D section, at depth H_1' from the surface.
- The effective stress at various section are determined as follows:-



Section A-A

$\sigma = 0;\ u = 0,\ \therefore \bar{\sigma} = 0$

Section D-D

$\sigma = \gamma_1 \cdot H_1'$; $u = 0$ (because piezometer shows no reading at D-D section, because there is no water above.)

$\therefore \bar{\sigma} = \sigma - u$

$= \gamma_1 H_1' - 0 = \gamma_1 H_1'$ where, γ_1 is unit weight of soil above D-D section.

Section B-B:-

$\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1''$ and $u = \gamma_w \cdot H_1''$

$\bar{\sigma} = \gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1'' - \gamma_w \cdot H_1''$

$= \gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1'' - \gamma_w \cdot H_1''$

$= \gamma_1 H_1' + H_1'' [(\gamma_{sat})_1 - \gamma_w]$

$= [\gamma_1 H_1' + H_1'' \gamma_1']$ where γ_1' is the submerged unit weight of the soil-1.

Section C-C

$\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1'' + (\gamma_{sat})_2 \cdot H_2$

$u = \gamma_w (H_2 + H_1'')$

$\therefore \bar{\sigma} = (\sigma - u) = [\gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1'' + (\gamma_{sat})_2 \cdot H_2] - \gamma_w (H_2 + H_1'')$

$= \gamma_1 \cdot H_1' + (\gamma_{sat})_1 \cdot H_1'' + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2 - \gamma_w H_1''$

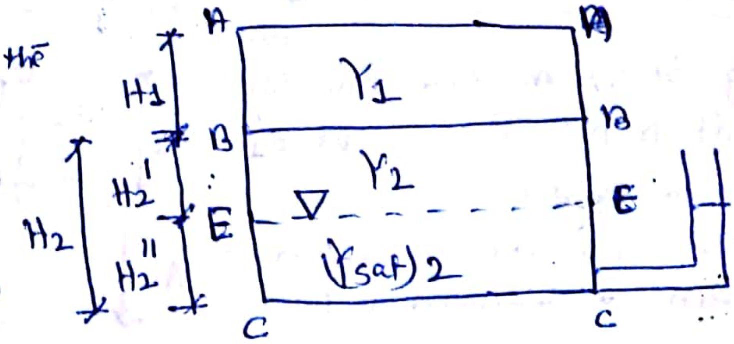
$= \gamma_1 \cdot H_1' + (\gamma_{sat})_1 \cdot H_1'' - \gamma_w H_1'' + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2$

$= \gamma_1 \cdot H_1' + H_1'' ((\gamma_{sat})_1 - \gamma_w) + H_2 ((\gamma_{sat})_2 - \gamma_w)$

$= [\gamma_1 \cdot H_1' + H_1'' \cdot \gamma_1' + H_2 \cdot \gamma_2']$ [γ_2' is submerged unit weight of soil-2]

(6) (4) water table in soil-2 :-

- fig. shows the condition when the water table is in soil-2 at section EE at depth of H_2' from the section B-B.
- the effective stress at various sections are as follows:-



Section (A-A)

$\sigma = 0, u = 0, \therefore \bar{\sigma} = 0$

Section (B-B)

$\sigma = \gamma_1 \cdot H_1 \quad \therefore u = 0$

$\therefore \bar{\sigma} = \sigma - u = \gamma_1 H_1 - 0 = \gamma_1 \cdot H_1$

Section EE

$\sigma = (\gamma_1 \times H_1) + (\gamma_2 \cdot H_2') \quad ; \quad u = 0$

$\therefore \bar{\sigma} = \sigma - u = (\gamma_1 H_1) + (\gamma_2 H_2') - 0$

$= (\gamma_1 H_1) + (\gamma_2 H_2') = [\gamma_1 H_1 + \gamma_2 H_2']$

section C-C

$\sigma = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 \cdot H_2''$

$u = \gamma_w \times H_2''$

$\therefore \bar{\sigma} = (\sigma - u) = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 \cdot H_2'' - \gamma_w \cdot H_2''$

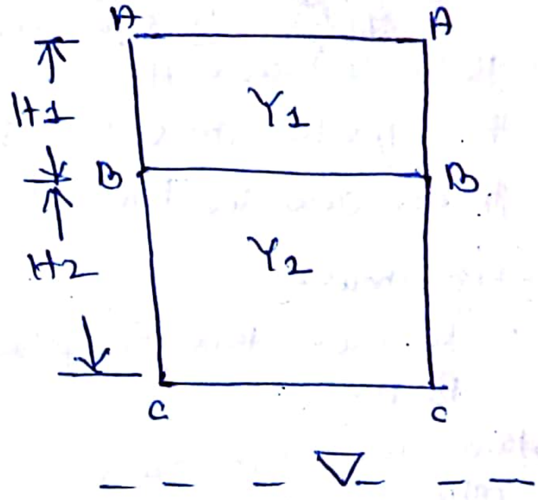
$= \gamma_1 H_1 + \gamma_2 H_2' + H_2'' [(\gamma_{sat})_2 - \gamma_w]$

$= \gamma_1 H_1 + \gamma_2 H_2' + H_2'' \cdot \gamma_2'$

$= \gamma_1 H_1 + \gamma_2 H_2' + H_2'' \cdot \gamma_2'$ where, γ_2' is the submerged unit weight of soil-2.

(5) Water Table below - C-C

- Fig shows the condition when the water table is below C-C.
- As the pore pressure is zero everywhere, the effective stresses are also equal to the total stress.



Section

- Effective stress at different section is calculated below:-

Section A-A

$$\sigma = 0, \quad u = 0, \quad \bar{\sigma} = 0.$$

Section B-B

$$\sigma = \gamma_1 H_1, \quad u = 0$$

$$\therefore \bar{\sigma} = \sigma - u = \gamma_1 H_1 - 0 = \gamma_1 \cdot H_1.$$

Section - C-C

$$\sigma = (\gamma_1 H_1 + \gamma_2 H_2)$$

$$u = 0$$

$$\therefore \bar{\sigma} = \sigma - u = (\gamma_1 H_1 + \gamma_2 H_2) - 0$$

$$\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2$$

Notes:-

Above the experiments we can say:-

(1) The effective stress at any section goes on increasing as the water table goes down.

(2) The effective stress depends upon the bulk unit weight above the water table and submerged unit weight below the water table.

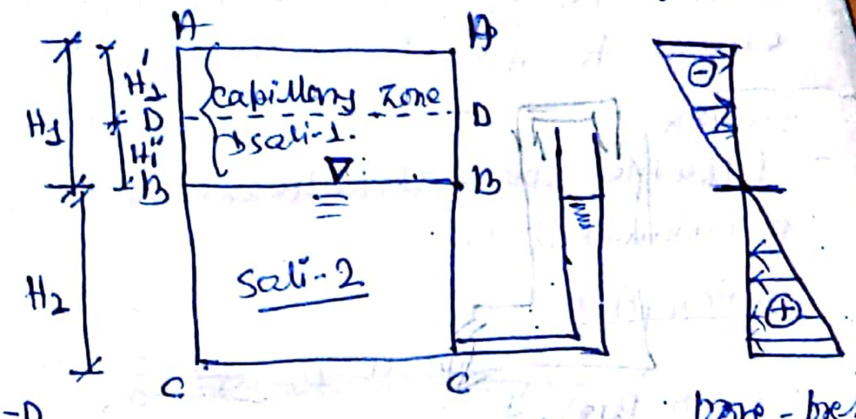
(3) The effective stress in a soil mass can be determined from the basic definitions without memorising any formula.

5) # compute effective stress for the static ground water conditions:-
⇒ effective stress in soils saturated by capillary action

- {If the soil above the water table is saturated by capillary action the effective stress can be determined using the basic concept.
- In this case the pore water pressure above the water table is negative.

- Fig: shows.
  - The water table is at level B-B.

⇒ two cases may arise.
(a) Soil saturated upto surface level A-A.
(b) Soil saturated upto level D-D



(1) Soil- saturated upto surface level A-A

- the pore pressure diagram is drawn on the right side.
- the stresses at various sections are determined as under.

(a) - Section A-A :-

$$\sigma = 0, \quad u = -\gamma_w H_1$$

(note; the height of capillary is taken from upper part of water level so H_1 is taken.]

$$\therefore \bar{\sigma} = \sigma - u$$
$$= 0 - (-\gamma_w H_1) = \gamma_w H_1$$

- If the soil was not saturated with capillary action, the effective stress at section A-A would have been zero.
- the capillary action has increased the effective stress by $\gamma_w H_1$.
[The negative pressure acts like a surcharge (q)]

at Section D-D :-

(height taken from pore pressure diagram)

$$\sigma = (\gamma_{sat})_1 \cdot H_1' \quad ; \quad u = -\gamma_w \cdot H_1'' = -\gamma_w (H_1 - H_1')$$

$$\therefore \bar{\sigma} = (\sigma - u) = [(\gamma_{sat})_1 \cdot H_1' - [-\gamma_w (H_1 - H_1')]]$$
$$= [(\gamma_{sat})_1 \cdot H_1' + \gamma_w (H_1 - H_1')]$$

$$\left[(\gamma_{sat})_1 \cdot H_1' + \gamma_w H_1 - \gamma_w H_1'\right]$$

$$= \left[(\gamma_{sat})_1 \cdot H_1' - \gamma_w H_1' + \gamma_w H_1\right]$$

$$= H_1'\left[(\gamma_{sat})_1 - \gamma_w\right] + \gamma_w H_1$$

$$= H_1' \cdot \gamma_1' + \gamma_w H_1 \rightarrow$$ increased due part due to capillary action.

Note:- If the soil had been saturated due to rise in water-table to A-A, the effective stress at section D-D would have been $\gamma_1' H_1'$. Thus the effective stress is increased by $(\gamma_w H_1)$ due to capillary action.

Section-B-B

$$\sigma = (\gamma_{sat})_1 \cdot H_1, \quad u = 0 \text{ (from pore pressure diagram.)}$$

$$\bar{\sigma} = (\sigma - u) = \left[(\gamma_{sat})_1 \cdot H_1 - 0\right] = \left[(\gamma_{sat})_1 \cdot H_1\right]$$

$$= \left[(\gamma_1' + \gamma_w) \cdot H_1\right] = \left[\gamma_1' \cdot H_1 + \gamma_w \cdot H_1\right]$$

If the soil above B-B had been saturated due to rise in water table to ~~A-A~~ A-A, the effective stress would have been $\gamma_1' H_1$. Thus the effective stress is increased by $\gamma_w H_1$ by capillary action.

Section-C-C

$$\sigma = (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 \; : \; u = \gamma_w H_2$$

in pore pressure diagram the pressure at depth H_2 below W.T. is considered (+) and also when the piezometer inserted gives the value $\gamma_w H_2$

Therefore,

$$\bar{\sigma} = (\sigma - u) = (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 - u$$

$$= (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_2$$

$$= (\gamma_{sat})_1 \cdot H_1 + H_2\left[(\gamma_{sat})_2 - \gamma_w\right]$$

$$= (\gamma_{sat})_1 \cdot H_1 + H_2 \gamma_2'$$ where, γ_2' is submerged unit weight of soil-2

(10)

$\bar{\sigma} = (\gamma_1' + \gamma_w) H_1 + \gamma_2' H_2$.

$= \gamma_1' H_1 + \underset{\sim}{\gamma_w H_1} + \gamma_2' H_2$ where, γ_1' is the submerged unit of Soil-1.

Thus, the effective stress increased due to capillary action by $(\gamma_w H_1)$

- It may be noted that the effective stress at all levels below the plane of saturation A-A, due to capillary water, is increased by $(\gamma_w H_1)$.
- The capillary water pressure $(\gamma_w H_1)$ acts as if a surcharge.

(2) Soil Saturated upto level D-D :-

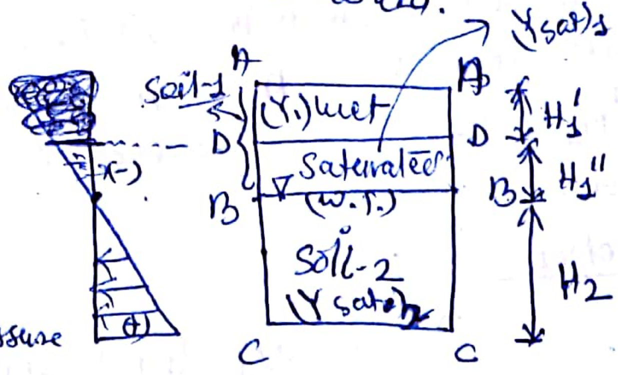
- Let us consider the case when the soil above the water table B-B is saturated only upto level D-D upto a height H_1''.
- The soil above level D-D is wet and has a unit weight of (γ_1)
- The capillary rise in this case is H_1''.

The stress at various sections can be determined as usual.

Section - A-A

$\sigma = 0, \quad u = 0, \quad \bar{\sigma} = 0$.

$\therefore$ no effect of capillary water.



Section - D-D

$\sigma = \gamma_1 \cdot H_1', \quad u = -\gamma_w H_1''$.

(from pore pressure diagram)

$\bar{\sigma} = \sigma - u$

$= \gamma_1 \cdot H_1' - (-\gamma_w H_1'')$

$= \gamma_1 \cdot H_1' + \gamma_w H_1''$

effective stress due to capillary pressure is increased by $(\gamma_w H_1'')$.

Section-B-B

$$\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1''$$
$$= \gamma_1 H_1' + (\gamma_1' + \gamma_w) H_1''$$
$$= \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1''$$

$u = 0$ (from the pore pressure diagram.) [This section lies at the point from where the water is sucked up so the pressure at this point or line is zero.]

$$\bar{\sigma} = \sigma - u$$
$$= (\gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1'') - 0$$
$$= [\gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1''.] \text{ Ans}$$

Section-C-C

$$\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 \cdot H_1'' + (\gamma_{sat})_2 \cdot H_2$$
$$= \gamma_1 H_1' + (\gamma_1' + \gamma_w) H_1'' + (\gamma_2' + \gamma_w) \cdot H_2$$
$$= \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1'' + \gamma_2' H_2 + \gamma_w \cdot H_2$$

$u = + \gamma_w \cdot H_2$ (from pore pressure diagram)

$$\bar{\sigma} = \sigma - u$$
$$= (\gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1'' + \gamma_2' H_2 + \gamma_w \cdot H_2) - \gamma_w \cdot H_2$$
$$= \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1'' + \gamma_2' H_2 + \cancel{\gamma_w \cdot H_2} - \cancel{\gamma_w \cdot H_2}$$
$$= \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_w H_1'' + \gamma_2' H_2$$

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points:-

- The capillary water above the water table causes a negative pressure ($\gamma_w H$) where H is the capillary rise.
- The negative pressure causes an increase in the effective stre at all levels below the saturation level. The increase is equal to "$\gamma_w H$". The capillary action is equivalent to a surcharge $q = \gamma_w H$

⇒ If the soil is saturated due to rise in water table; the effective stress depends upon the submerged unit weight, whereas; for the soil saturated with capillary water, the effective stress depends upon the "saturated unit weight".

- In the capillary water phenomenon; the water does not contribute to hydrostatic pressure.
- If the water table rises to the top soil surface, the meniscus is destroyed and the capillary water changes to the free water and the effective stress is reduced throughout.

→ pore water pressure in the capillary zone is negative (-ve)

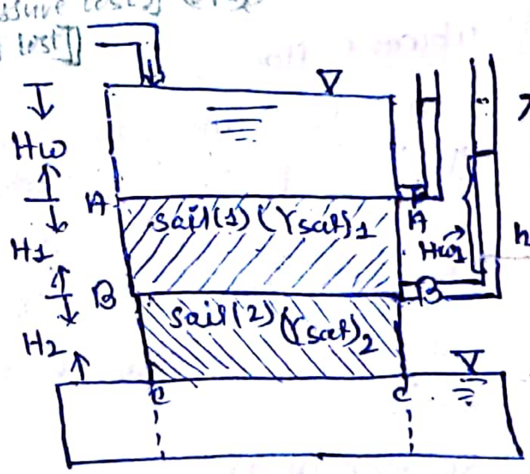
Effective stresses under steady seepage conditions: → when the water flows through the soil, it exerts a seepage force on the soil particles.

- seepage force affects the interparticle forces and hence the effective stress.
- effective stress is increased when the flow is downward, as the seepage force increases the interparticles force.
- effective stress is decreased when the flow is upward; as the seepage force decreases the interparticle forces.

Two cases are discussed saperately:-

(a) **Down-ward flow:**- [[u = static pressure − total dynamic pressure lost]] (OR) [[h_w = static head − total dynamic head lost]]

- Let us consider the case when the flow is downward.
- The head causing flow is "h"
- the pore water pressure at sections (A-A) and (B-B) are indicated by the piezometers.
- the effective stress at various section are determined as follows:-



Section A-A: →

$$\sigma = \gamma_w H_w, \quad u = \gamma_w H_w$$

$$\therefore \bar{\sigma} = (\sigma - u) = \gamma_w H_w - \gamma_w H_w = 0$$

Section B-B: →

$$\sigma = (\gamma_{sat})_1 \times H_1 + \gamma_w H_w \quad ; \quad u = \gamma_w H_{w1} \quad \text{where, } (H_w + H_1) > H_{w1}$$

therefore,

$$\bar{\sigma} = (\sigma - u) = (\gamma_{sat})_1 \cdot H_1 + \gamma_w H_w - \gamma_w H_{w1}$$

$$= (\gamma_1' + \gamma_w) H_1 + \gamma_w H_w - \gamma_w H_{w1}$$

$$= \gamma_1' H_1 + \gamma_w H_1 + \gamma_w H_w - \gamma_w H_{w1}$$

$$\bar{\sigma} = \gamma_1' H_1 + \gamma_w (H_1 + H_w - H_{w1})$$

$$= \gamma_1' H_1 + \gamma_w \underbrace{(H_w + H_1 - H_{w1})}_{\text{positive, because, } (H_w + H_1) > H_{w1}}$$

∴ Effective stress increased due to down-ward flow.

(i) section- c-c

$$\sigma = \gamma_w H_w + (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 \quad ; \quad u = 0$$

therefore,-

$$\bar{\sigma} = \gamma_w H_w + (\gamma_1' + \gamma_w) \cdot H_1 + (\gamma_2' + \gamma_w) H_2$$

$$\text{or, } \bar{\sigma} = \gamma_w H_w + \gamma_1' H_1 + \gamma_w \cdot H_1 + \gamma_2' H_2 + \gamma_w H_2$$

$$\text{or, } \bar{\sigma} = \gamma_1' H_1 + \gamma_2' H_2 + \gamma_w H_w + \gamma_w H_1 + \gamma_w H_2$$

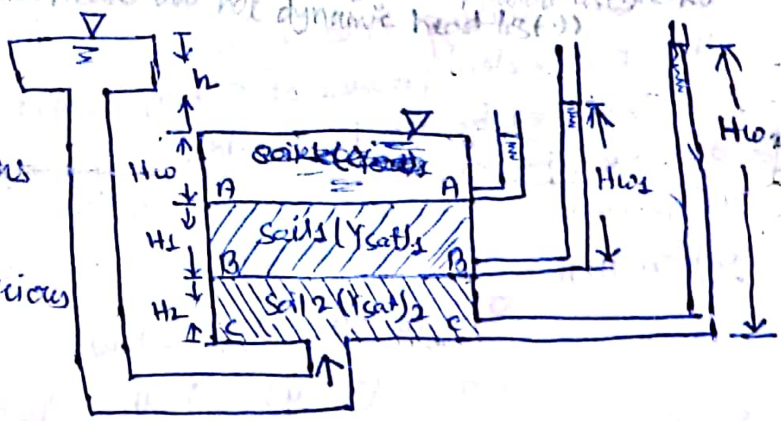
$$= \gamma_1' H_1 + \gamma_2' H_2 + \gamma_w (H_w + H_1 + H_2)$$

$$= \gamma_1' H_1 + \gamma_2' H_2 + \gamma_w h.$$

A comparison with the effective stresses corresponding to hydrostatic conditions shows that the effective stress is increased by "$\gamma_w h$"

(b) upward flow:→

- fig. show the case when the flow is upward; direction.
- the piezometers at various elevations indicate the pore water pressure.
- calculation of effective stress at various sections =



upward flow:-

Section (A-A)

$$\sigma = \gamma_w H_w \quad \therefore \; u = \gamma_w H_w$$

$$\bar{\sigma} = (\gamma_w H_w - \gamma_w H_w) = 0$$

Section (B-B)

$$\sigma = \gamma_w H_w + (\gamma_{sat})_1 \cdot H_1 \; ; \quad u = \gamma_w H_{w1} \text{ where, } H_{w1} > (H_1 + H_w)$$

$$\therefore \bar{\sigma} = (\sigma - u) = \gamma_w H_w + (\gamma_{sat})_1 \cdot H_1 - \gamma_w H_{w1}$$

$$= \gamma_w H_w + (\gamma_1' + \gamma_w) H_1 - \gamma_w H_{w1}$$

$$= \gamma_w H_w + \gamma_1' H_1 + \gamma_w H_1 - \gamma_w H_{w1}$$

$$= \gamma_1' H_1 + \gamma_w H_w + \gamma_w H_1 - \gamma_w H_{w1}$$

$$= \gamma_1' H_1 + \gamma_w (H_w + H_1 - H_{w1})$$

Since, $H_{w1} > (H_w + H_1)$ ∴ the term $(H_w + H_1 - H_{w1})$ is negative.

So, the effective stress decreases than the ~~caso~~ corresponding hydrostatic condition.

Section c-c

$$\sigma = \gamma_w H_w + (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 \,; \quad u = \gamma_w H_{w2}$$

$$\bar{\sigma} = (\sigma - u) = \gamma_w H_w + (\gamma_{sat})_1 \cdot H_1 + (\gamma_{sat})_2 \cdot H_2 - \gamma_w H_{w2}$$

$$= \gamma_w H_w + (\gamma_1' + \gamma_w) H_1 + (\gamma_2' + \gamma_w) H_2 - \gamma_w H_{w2}$$

$$= \gamma_w H_w + \gamma_1' H_1 + \gamma_w H_1 + \gamma_2' H_2 + \gamma_w H_2 - \gamma_w H_{w2}$$

$$= \gamma_1' H_1 + \gamma_2' H_2 + \gamma_w H_w + \gamma_w H_1 + \gamma_w H_2 - \gamma_w H_{w2}$$

$$= \gamma_1' H_1 + \gamma_2' H_2 + \gamma_w (H_w + H_1 + H_2 - H_{w2})$$

$$= \gamma_1' H_1 + \gamma_2' H_2 - \gamma_w (H_{w2} - H_w - H_1 - H_2)$$

$$= \gamma_1' H_1 + \gamma_2' H_2 - \gamma_w (h)$$

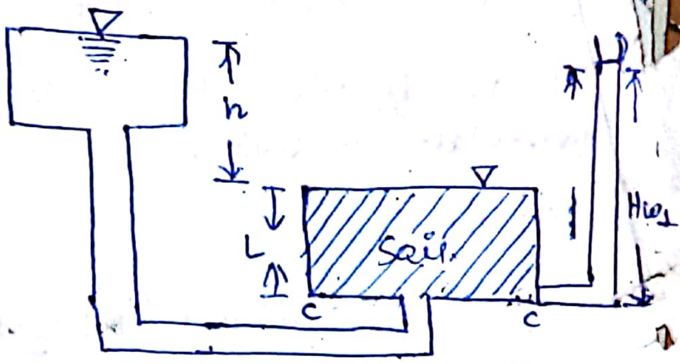
∴ Thus the effective stress is reduced by "$\gamma_w h$" from the corresponding hydrostatic conditions.

Quick-sand conditions:⇒

- the effective stress is reduced due to upward flow of water.
- when the head causing upward flow is increased, the stage is eventually reached when the effective stress is reduced to zero.
- The condition so developed is called the quick-sand condition.

(18) - figure shows a soil specimen of length "L" subjected to upward pressure.

- Let us consider the stresses developed at section (c-c)



Section (c-c)

$\sigma = (\gamma_{sat} L) = (\gamma' + \gamma_w) L$

$u = \gamma_w H_{w_1} = \gamma_w (L+h)$

$\bar{\sigma} = (\sigma - u) = (\gamma' + \gamma_w) L - \gamma_w (L+h)$

$= (\gamma' + \gamma_w) L - \gamma_w L - \gamma_w h$

$= \gamma' L + \cancel{\gamma_w L} - \cancel{\gamma_w L} - \gamma_w h$

$= \gamma' L - \gamma_w h = (\gamma' L - \gamma_w h)$

Second term can be written in terms of hydraulic gradient as under:-

$\gamma_w h = \gamma_w \times (h/L) \times L = \gamma_w \cdot i \cdot L$

$\therefore \bar{\sigma} = \gamma' L - \gamma_w i L$

the effective stress become zero if:

$\gamma' \cancel{L} = \gamma_w i \cancel{L}$

or, $\boxed{i = \gamma' / \gamma_w}$

thus, the hydraulic gradient at which the effective stress becomes zero is known as critical gradient (i_c).

Thus; $i_c = \gamma' / \gamma_w$.

∴ Substituting the value of submerged unit weight in terms of void ratio :. $\left(i.e., \gamma' = \left(\frac{G-1}{1+e}\right) \cdot \gamma_w \right)$

$$i_c = \left(\frac{G-1}{1+e}\right)$$

taking the specific gravity of soils (G) as 2.67 and void ratio (e) = 0.67

$$i_c = \frac{2.67-1}{1+0.67} = 1.0$$

thus, the effective stress becomes zero for the soil with above values of "G" and "e". When the hydraulic gradient is unity i.e, head causing flow is equal to the length of the specimen.

Effect of Surcharge and Submergence on quick condition :-

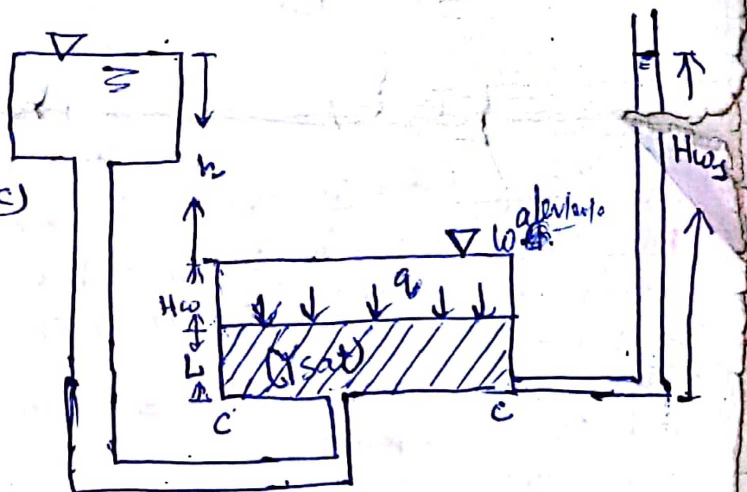
- Fig shows a soil specimen submerged under water and subject to surcharge load of intensity "q".
- Let us consider the stresses at section (C-C)

Now,

$$\sigma = \gamma_w H_w + q + (\gamma_{sat}) L$$

$$u = \gamma_w H_{w1}$$

$$= \gamma_w (h + H_w + L)$$



Effect of Surcharge:

$$\bar{\sigma} = (\sigma - u)$$

$$= \gamma_w H_w + q + (\gamma_{sat}) L - \gamma_w (h + H_w + L)$$

the soil will become to quick ~~sand~~ condition when, $\bar{\sigma} = 0$. Thus-

$$\bar{\sigma} = \gamma_w H_w + q + (\gamma_{sat}) L - \gamma_w (h + H_w + L) = 0$$

or, $\gamma_w H_w + q + (\gamma_{sat}) L - \gamma_w (h + H_w + L) = 0$

or, $\gamma_w H_w + q + (\gamma_{sat}) L = \gamma_w (h + H_w + L)$

(22) or, $\gamma_w h + \gamma_w H_w + \gamma_w L = \gamma_w H_w + q + (\gamma_{sat}) L$

or, $\gamma_w h + \gamma_w L = q + (\gamma_{sat}) L$

or, $\gamma_w h = q + (\gamma_{sat}) L - \gamma_w L$

$= q + L(\gamma_{sat} - \gamma_w)$

or, $\gamma_w h = q + L\gamma'$

or, $h = \dfrac{q + L\gamma'}{\gamma_w}$

Substituting $q = 0$,

$h = \dfrac{0 + L\gamma'}{\gamma_w}$ or, $\dfrac{h}{L} = \gamma'/\gamma_w$

or, $\boxed{i = \gamma'/\gamma_w}$

* Failures of hydraulic structure by piping:-

Piping failure → structure built on the pervious foundation sometimes fail by formation of pipe shaped channel in its foundation.

-- failure occurs when the water flowing through the foundation has a very high hydraulic gradient and it carries soil particles.

→ Two types of such failure :- ① Back-ward-erosion piping failure.

② Heave-piping failure :→ occurs in the form of a heave (rise) of a large mass of soil due to upward seepage force. When hydraulic gradient at exit exceeds the critical gradient (i.e. lifting force due to upward flow of water at any level is greater than the submerged weight of the soil above that level) the entire soil mass then heaves up and is blown out by the percolating water.

Prevention of piping failures:-

① Increase the path of percolation - by increasing the path of percolation mean increasing the "L" so the hydraulic gradient will come to less value and it will safe.

② Reducing seepage :- (seepage is reduced by providing impervious core.

③ Providing drainage filter :- (drainage is provided which change the direction of flow away from downstream.

④ Loaded filter ...